Commutation relations(using Index Notation)

- $[A, BC] = [A, B]\,C + B\,[A, C]$
- $[p_j, x_k] = -i\hbar\delta_{jk}$
- $[x_a p_j, x_k] = -i\hbar x_a \delta_{jk}$

$$\begin{aligned}[x_a p_j, x_k] &= x_a\,[p_j, x_k] + [x_a, x_k]\,p_j \\ &= x_a(-i\hbar\delta_{jk}) \\ &= -i\hbar\delta_{jk}x_a\end{aligned}$$

- $[x_a p_j, p_k] = -i\hbar\delta_{jk}p_a$
- $[p^n, q] = -i\hbar n p^{n-1}$ This is easily shown by induction

$$\begin{aligned}\left[p^2, q\right] = ppq - qpp &= p(-i\hbar + qp) - qpp \\ &= -i\hbar p + pqp - qpp \\ &= -i\hbar p + [p, q]\,p \\ &= -2i\hbar p\end{aligned}$$

  And so on. Each iteration adds a factor of $-i\hbar p$

- $[q^n, p] = i\hbar n q^{n-1}$
- $[L_b, x_k] = i\hbar\epsilon_{bka}x_a$

$$\begin{aligned}[L_b, x_k] &= [\epsilon_{baj}x_a p_j, x_k] \\ &= \epsilon_{baj}\,[x_a p_j, x_k] \\ &= \epsilon_{baj}(-i\hbar x_a\delta_{jk}) \\ &= -i\hbar\epsilon_{bak}x_a \\ &= i\hbar\epsilon_{bka}x_a\end{aligned}$$

- $[L_z, x + iy] = \hbar(x + iy)$
- $[L_z, x - iy] = \hbar(x - iy)$
- $[L_b, p_k] = i\hbar\epsilon_{bka}p_a$
- $[L_b, L_k] = i\hbar\epsilon_{bka}L_a$
- $\left[L^2, L_i\right] = 0$

$$\begin{aligned}\left[L^2, L_i\right] &= [L_j L_j, L_i] \\ &= L_j\,[L_j, L_i] + [L_j, L_i]\,L_j \\ &= \epsilon_{ijk}L_j L_k + \epsilon_{ijk}L_k L_j \\ &= \epsilon_{ijk}L_j L_k - \epsilon_{ijk}L_j L_k \\ &= 0\end{aligned}$$

- $[\mathbf{L.S}, L_z] = i\hbar(L \times S)_z$

$$\begin{aligned}
[\mathbf{L.S}, L_z] &= L_i \left[S_i, L_z\right] + \left[L_i, L_z\right] S_i \\
&= 0 + (i\hbar\epsilon_{izk} L_k) S_i \\
&= i\hbar\epsilon_{zki} L_k S_i \\
&= i\hbar(L \times S)_z
\end{aligned}$$

- $[\mathbf{L.S}, S_z] = i\hbar(L \times S)_z$

$$\begin{aligned}
[\mathbf{L.S}, S_z] &= L_i \left[S_i, S_z\right] + \left[L_i, S_z\right] S_i \\
&= i\hbar\epsilon_{izk} L_i S_k) + 0 \\
&= -i\hbar(L \times S)_z
\end{aligned}$$

- $[\mathbf{L.S}, L_z + S_z] = [\mathbf{L.S}, J_z] = 0$

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$$\begin{aligned}
\mathbf{p}_{mech} &= \mathbf{p}_{can} - q\mathbf{A} \\
\left[\mathbf{p}_{mech-i}, \mathbf{p}_{mech-j}\right] &= \left[\mathbf{p}_i - q\mathbf{A}_i, \mathbf{p}_j - q\mathbf{A}_j\right] \\
&= \left[\mathbf{p}_i, \mathbf{p}_j\right] + q^2 \left[\mathbf{A}_i, \mathbf{A}_j\right] - q \left[\mathbf{p}_i, \mathbf{A}_j\right] \\
&= 0 + 0 + i \left(\frac{\partial}{\partial i} \mathbf{A}_j - \frac{\partial}{\partial j} \mathbf{A}_i \right) \\
&= i\mathbf{F}_{ij} = i\epsilon^{ijk} B_k
\end{aligned}$$